FD-TD/Matrix-Pencil Method for the Efficient Simulation of Waveguide Components Including Structures of More General Shape

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Abstract - **A combined FD-TD/matrix-pencil method is introduced for the efficient and rigorous calculation of the full-wave modal S-parameters of waveguide components including structures of more general shape or of high complexity. The application of the S-parameter definition for unmatched ports requires merely standard Mur's absorbing boundaries for reliable results, and a nonorthogonal or contour path mesh formulation allows the convenient inclusion of curved boundaries. The efficiency of the method is demonstrated at the analysis of waveguide components of practical importance, such as the twisted waveguide, the twisted waveguide bend, and the waffle-iron filter. The proposed method is verified by excellent agreement with FEM/mode-matching results.**

I. Introduction

Several techniques have been applied in the past for analyzing waveguide structures of more general shape, such as miter compensated T-junctions, analyzed by a FEM method in [1], or arbitrarily shaped H- and E-plane discontinuities, which are investigated by a boundary-contour mode-matching method in [2]. Due to its high flexibility, the FD-TD method is considered to be particularly well applicable for the analysis of waveguide elements, which has been demonstrated recently, [3] -[6].

Typical elements hitherto investigated by the FD-TD method, however, are inductive irises or inductive iris filters, transitions and T-junctions, H-plane couplers, H-plane corners with inductive posts, and cylindrical cavities [3] - [6]. It indicates that mostly simple step type structures have been analyzed so far which have been calculated also by the mode matching technique before. This seems to be mainly due to well-known problems in the usual FD-TD simulation of more general waveguide elements, for instance the large number of required time steps, and the influence of the highly dispersive waveguide ports.

This paper presents an improved FD-TD-based approach which allows the efficient full-wave modal S-matrix calculation of a comprehensive class of more general or complicated waveguide structures, such as bends, twisted bends, and waffle-iron filters (Fig.1). The hitherto existing typical problems in the usual FD-TD simulation of waveguide elements are solved successfully by adequate techniques: 1) For the first time, the very efficient matrix pencil technique [7] is utilized for the FD-TD method. This reduces the number of involved time steps significantly. The matrix pencil technique requires less numerical effort than, for instance, the often used Prony's method [8]. 2) The application of the modal S-parameter definition for unmatched ports [9] achieves even with standard Mur's absorbing boundaries excellent and reliable results also for the higher-order modes. 3) A structure dependent mesh is used based on nonorthogonal or contour path grid cells, respectively, according to the specific form of the boundary.

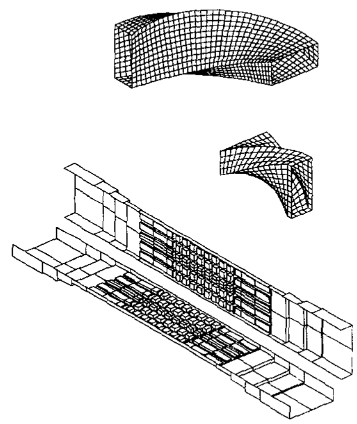

Fig. 1: Typical structures investigated with the improved FD-TD technique: Twisted 90°-waveguide, twisted 90°-bend, and waffle-iron filter.

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II. Theory

A. Matrix Pencil Technique

Like for Prony's method [8], also for the matrix pencil technique the time transient wave form is approximated [7] by a sum of damped complex exponentials

$$\begin{aligned} y_k &= x_k + n_k = \sum_{t=1}^{M} |b_t| e^{(\alpha_t + j\omega_t)k + j\phi_t} + n_k \\ &= \sum_{t=1}^{M} b_t z_t^k + n_k, \end{aligned} \tag{1}$$

where $k = 0, 1, \ldots, N-1$ is the time index, n_k indicates additional noise. The basic idea of the matrix pencil method – and the significant difference in comparison with Prony's method – is to formulate an eigenvalue problem [7] for the determination of the poles $z_t,\ t = 1, \ldots, M$.

With the data vectors $\boldsymbol{x_t}$ of length $N - L$ for the noiseless signal x_k

$$\boldsymbol{x}_t = [x_t, x_{t+1}, \ldots, x_{N-L+t-1}]^T \ . \tag{2}$$

the matrices X_0 and X_1 are defined

$$\underset{(N-L)\times L}{X_0} = [\boldsymbol{x}_{L-1}, \boldsymbol{x}_{L-2}, \ldots, \boldsymbol{x}_0] \tag{3}$$

$$\underset{(N-L)\times L}{X_1} = [\boldsymbol{x}_{L}, \boldsymbol{x}_{L-1}, \ldots, \boldsymbol{x}_1] \, . \tag{4}$$

If the 'pencil-parameter' L is chosen to be $M \leq L \leq N - M$, the matrix pencil $X_1 - z_t X_0$ is of rank $M - 1$, and each pole z_t is one of the M nonzero eigenvalues of the generalized eigenvalue problem

$$(X_1 - zX_0)\, \boldsymbol{q} = 0. \tag{5}$$

Eq. (5) is transformed into a standard eigenvalue problem by multiplication from left with the pseudo inverse [10] X_0^+. For the noisy signal, the data matrices Y_0, Y_1, defined analogously to X_0, X_1, might have the full rank, even if the signal contains only $M < min(N - L, L)$ poles. Therefore, the transformation of (5) into a standard eigenvalue problem of a matrix of rank M is performed by the multiplication with the 'truncated pseudo inverse matrix' Y_0^+ [10]. To obtain an estimation of the number of poles M of the noisy signal, and to compute Y_0^+, a singular value decomposition [11] of $Y_0 = U\Sigma V^H$ is carried out. After computing the poles z_t, i.e. the eigenvalues of $(Y_0^+ Y_1 - zI)\boldsymbol{q} = 0$, the residues b_t are obtained easily by solving a least square problem.

B. Modal S-parameter extraction

The proposed technique is based on the general modal S-parameter definition in the case of unmatched ports, i.e. the structure is simulated with non-ideal absorbing boundaries. This implies that on each port both incident and scattered propagating waves (and/or evanescent modes) appear, even if only one port has been excited. In the case of a general N-port discontinuity, we have to consider a system of N equations in the form:

$$B = SA, \tag{6}$$

where $B = (\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_N)$ is a matrix formed by N different $\boldsymbol{b}$-vectors, and $A = (\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_N)$ is a matrix formed by N different **a**-vectors. The desired modal S-matrix is then obtained by the multiplication of equation (6) with the inverse of matrix A, from the right side.

The N different vectors **a** and **b**, respectively, are calculated by N appropriate simulation runs, each considering a different condition, e.g. the excitation of a different port. If more then one mode are present on a port of the structure under investigation, an extraction of the modal guided power has to be performed. For that purpose, we use the orthogonal mode properties.

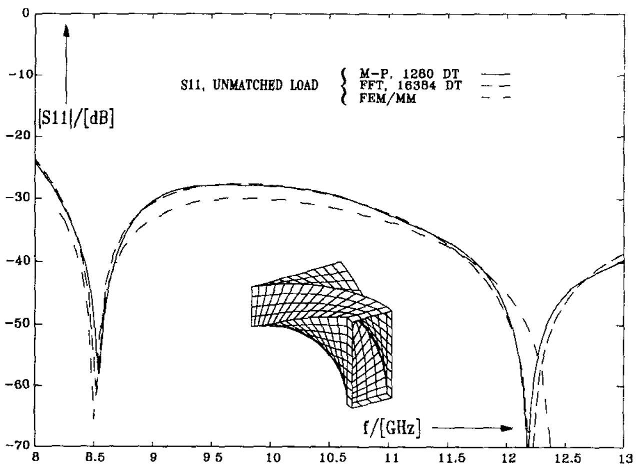

Fig. 2: a) Return loss of a 90° twisted rectangular X-band waveguide. FD-TD-Matrix-Pencil-method, 1280 Δt (———), comparison with FD-TD-FFT, for 16384 Δt (– – –) and FEM/MM results [12] (- · - ·).

III. Results

For the application of the matrix pencil technique to the examples in this chapter, the number of signal poles for the calculation of the pseudo inverse matrix Y_0^+ is determined by the chosen minimum value of $\sigma_M \geq \sigma_1 \cdot 10^{-6}$. In all investigated cases, the whole time interval for the FD-TD simulation is chosen to be four times the value which an exited wave needs to cross the structure under investigation. Since the FD-TD time signals are very oversampled, only every $(N_{\text{skip}} + 1)$th value is considered for the matrix pencil formulation. The number of skipped time steps N_{skip} has to be selected so that the estimated number of poles M is less than the half of the totally involved time steps N_{MP}.

The first example is a twisted rectangular X-band waveguide, Fig. 2a and 2b. Very good agreement with own finite element/mode-matching (FE/MM) calculations [12] is shown by using the described modal unmatched-port S-parameter technique, Fig. 2a. In contrast to the application of a 16384-FFT, merely the first 1280 time iterations are required by using the matrix pencil technique, and N_{skip}=1 was used. Fig. 2b shows a comparison of the matrix-pencil results with DFT-results, obtained with only 1280 time steps.

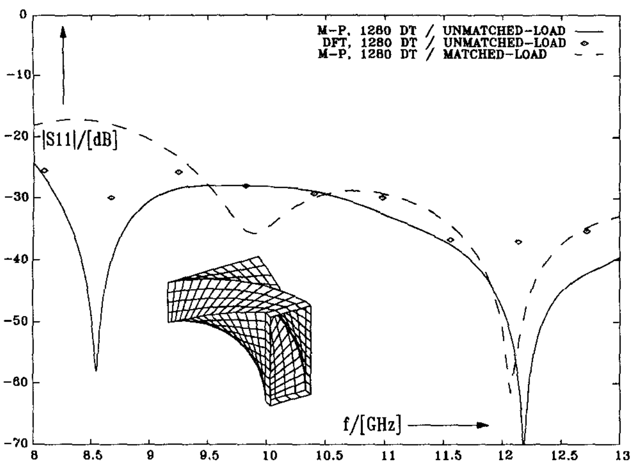

Fig. 2: b) Return loss of a 90° twisted rectangular X-band waveguide. Comparison with FD-TD-DFT results (◇ ◇) for 1280 Δt and the standard matched-load S-parameter extraction procedure (– · – ·) . Dimensions: WR90 waveguide, 22.86 × 10.16mm, length of the twisted region: 31.75mm. Applied discretization: 38 × 17 × 104 cells.

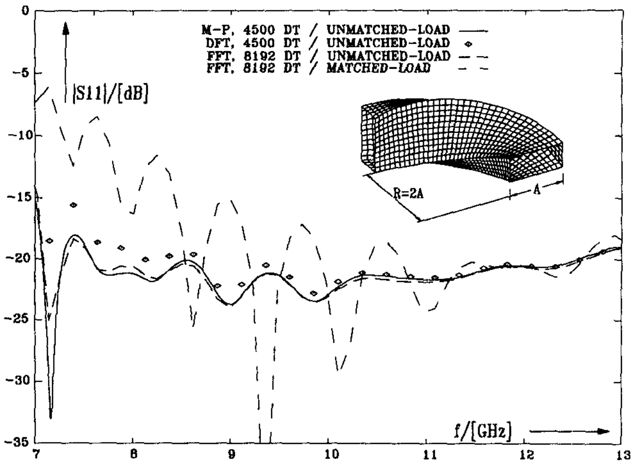

Fig. 3: Return loss of a 90° twisted 90°-bend, obtained by the FD-TD-Matrix-Pencil-method, 4500 Δt (———), compared with FD-TD-FFT results for 16384 Δt (– – –), FD-TD-DFT results for 4500 Δt (◇ ◇), and FD-TD-FFT results (– · – ·) utilizing the matched-load S-parameter extraction procedure. Dimensions: WR90 waveguide, A=22.86 × 10.16mm, bend-radius: 2A. Discretization: 34 × 16 × 202 cells.

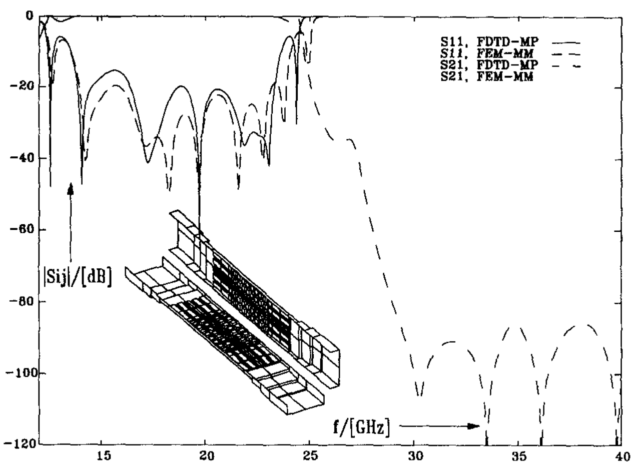

Fig. 4: Waffle-iron filter of Matthaei, Young, Jones. Verification of the FD-TD-Matrix-Pencil results (———), (– · · – ·) with the FE/MM method [12] (– – –), (· · · · ·).

The next example is a waffle-iron filter with the dimensions given in [13]. Good agreement with the FE/MM [12] results may be stated again, cf. Fig. 4.

IV. Conclusion

A very efficient FD-TD technique is introduced for the analysis of waveguide structures of nearly arbitrary shape and of high complexity. The involved matrix pencil technique requires less numerical effort than the often used Prony's method. The direct use of the modal S-parameter definition for unmatched ports, and a structure dependent mesh based on nonorthogonal or contour path grid cells, achieve even with standard Mur's absorbing boundaries excellent and reliable results also for higher-order modes.

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